Computer Exercise: Week 7
Zoology 535, Ecosystem Analysis

This week we will analyze a model of dynamics of grazed grassland vegetation, introduced by Noy-Meir (1975). The model combines a quadratic growth curve for the grasses (V) with a sigmoid herbivory function (H). Noy-Meir worked on systems grazed by goats and sheep in the Middle East. He assumed that the herbivory level was controlled by decisions of pastoralists who would select patches based on grass density. In other words, they would behave like switching predators, so the sigmoid herbivory function was appropriate.

The model is

$$dV/dt = r\,V\,[1-(V/k)] - [(c\,H\,V^2)/(h^2 + V^2)]$$

where

V = vegetation biomass kg / ha

H = herbivore level, animals / ha

r = maximum growth rate of vegetation, 1/time

k = carrying capacity of the rangeland for vegetation, kg / ha

c = maximum grazing rate, kg / (animal ha time)

h = vegetation level where grazing is half the maximum rate, kg / ha

Computer work:

Plot the net rate of vegetation growth (dV/dt) versus V for different levels of H, using the program VegRatePlot.R. The blue curve is high H, red curve is intermediate, green curve is low H. Where are the equilibria, for each value of H? Which ones are stable?

"Ball-and-cup" diagrams are another way of visualizing multi-stable ecosystem models. The program VegPotentialPlot.R plots ball-and-cup diagrams for the same 3 herbivore levels used in VegRatePlot.R. Because it is not easy to see what is going on near zero, the program VegPotentialNearZero.R "zooms in" the ball-and-cup curves for low values of V. How do the hills and valleys of the ball-and-cup curves correspond to the equilibria seen in VegRatePlot.R?

The program VegSim.R solves the model for V, at an intermediate level of H, starting very near the unstable equilibrium. What is going on here?

The program VegEq.R plots the equilibria of V versus H. Unstable equilibria are red, stable equilibria are green or blue. Can you explain this diagram to the person next to you?

Feedback

1) Suppose that a grassland has 1 animal/hectare. At equilibrium, what will the grass biomass be, approximately?

2) Now suppose that the animal density is slowly raised to 3 animals/hectare. What will happen to the grass biomass?

3) The pastoralist now slowly raises the animal density to 6 animals/hectare. What happens to the grass biomass?

4) The pastoralist now hastily cuts the animal density in half, to 3 animals/hectare. What happens to the grass biomass?

5) At this point, what must be done to raise the grass biomass level above 100 kg / ha?

A Note on Ball and Cup Diagrams for Multi-Stable Ecosystems

"Ball and cup" diagrams are sometimes used to visualize the stability of multi-stable systems like this vegetation model. To calculate a ball and cup diagram, define a "potential function" F such that dF/dV = dV/dt. So in our case

$dF/dV = r\ V\ [1-(V/k)] - [(c\ H\ V^2)/(h^2 + V^2)]$

To construct a graph of F, integrate dF/dV over a gradient of V using the computer. For each point on the gradient V_i, integrate dF/dV from 0 to V_i to compute F_i. Then plot -Fi against V_i to see the ball-and cup diagram.

Reference

Noy-Meir, I., 1975, Stability of grazing systems: an application of predator-prey graphs. J. Ecology 63: 459-481.